

About Uniformly Hattori Spaces

A.A. Borubaev¹, B.E. Kanetov^{*2}

¹*Institute of Mathematics of the National Academy of Sciences under the Prezident of the Kyrgyz Republic*

²*Jusup Balasagyn Kyrgyz National University, Bishkek, Kyrgyzstan*

Abstract

This paper introduces and studies, for the first time, uniformly Hattori spaces. In particular, it highlights unsolved problems related to the theory of uniformly Hattori spaces.

Keywords: uniform space, uniform cover, uniformly Hattori space, Hattori space.

2010 MSC: 54E15, 54D20.

1. Introduction

Throughout this paper all uniform spaces are assumed to be Hausdorff, topological spaces are Tychonoff.

Let X be a nonempty set. For covers α and β of the set X , $x \in X$ and $H \subset X$, we have $\alpha \wedge \beta = \{A \cap B : A \in \alpha, B \in \beta\}$, $\alpha(x) = \bigcup \text{St}(\alpha, x)$, $\text{St}(\alpha, x) = \{A \in \alpha : A \ni x\}$, $\alpha(H) = \bigcup \text{St}(\alpha, H)$, $\text{St}(\alpha, H) = \{A \in \alpha : A \cap H \neq \emptyset\}$.

For covers α and β of the set X , the symbol $\alpha \succ \beta$ means that the cover α is a refinement of the cover β , i.e. for any $A \in \alpha$ there exists $B \in \beta$ such that $A \subset B$, and the symbol $\alpha * \succ \beta$ means that α is a strongly star refinement of β , i.e. for any $A \in \alpha$ there exists $B \in \beta$ such that $\alpha(A) \subset B$. The cover α is finitely-additive if $\alpha^\angle = \alpha$, where $\alpha^\angle = \{\bigcup \alpha_0 : \alpha_0 \subset \alpha \text{ is finite}\}$.

A uniformity on a nonempty set X is a family U of covers of X which satisfies the following conditions:

(U1) If $\alpha \in U$ and β is a cover of X such that $\alpha \succ \beta$, then $\beta \in U$.

(U2) If $\alpha_1, \alpha_2 \in U$, then there exists $\alpha \in U$ such that $\alpha \succ \alpha_1$ and $\alpha \succ \alpha_2$.

(U3) If $\alpha \in U$, then there exists $\beta \in U$ such that $\beta * \succ \alpha$.

(U4) For any two distinct points $x, y \in X$, there is an $\alpha \in U$ such that no member of α contains both x and y .

The covers from U are called uniform covers, and the pair (X, U) is called a uniform space.

A family $B \subset U$ is called a base of uniformity U , if for any $\alpha \in U$ there exists $\beta \in B$ such that $\beta \succ \alpha$.

A family of covers B of the set X is a base of some uniformity U on X iff the following conditions are satisfied:

(B1) For any pair $\beta_1, \beta_2 \in B$ there exists $\beta \in B$ such that β refines both β_1 and β_2 ;

(B2) For any $\beta \in B$ there exists $\gamma \in B$ such that γ is a strongly star refinement of β ;

(B3) $\bigcap \{\beta(x) : \beta \in B\} = \{x\}$ for any point $x \in X$.

A uniformity (in terms of entourages) on a nonempty set X is a family U of subsets of $X \times X$ which satisfies the following conditions:

*Corresponding author: bek210873@gmail.com

(D1) Each element of the family U contains the diagonal Δ , and $\cap\{V : V \in U\} = \Delta$.

(D2) If $V \in U$, then $V^{-1} \in U$.

(D3) For any $V \in U$ there exists $W \in U$ such that $W \circ W \subset V$.

(D4) If $V \in U$ and $W \in U$, then $V \cap W \in U$.

(D5) If $V \in U$ and $V \subset W$, then $W \in U$.

The pair (X, U) is called a uniform space defined in terms of entourages of the diagonal Δ .

A family $B \subset U$ is called a base of the uniformity U , if for any $V \in U$ there exists $W \in B$ such that $W \subset V$.

The least cardinal number that is the power of some base of the uniformity U is called its weight and is denoted by $w(U)$. A filter F is called a Cauchy filter in (X, U) , if $\alpha \cap F \neq \emptyset$ for any $\alpha \in U$. [2]; a uniform space (X, U) is called precompact if it has a base consisting of finite covers [2]; a uniform space (X, U) is called τ -bounded if it has a base consisting of covers cardinality $\leq \tau$ [2]; a uniform space (X, U) is called complete if every Cauchy filter in it converges [2]; a uniform space (X, U) is called sequentially complete if every Cauchy filter having a countable base in it converges [2]; a Tychonoff space X is called complete in Diedonne sense, if there exists a complete uniformity generating topology of the space X [2]; a uniform space (X, U) is called uniformly locally compact if there exists a uniform cover consisting of compact subsets [8]; a uniform space (X, U) is called uniformly A -paracompact in the sense of Aparina if every finitely-additive open cover of (X, U) has a locally finite uniform refinement [1]; a uniform space (X, U) is called strongly uniformly A -paracompact if every finitely-additive open cover of (X, U) has a star-finite uniform refinement [5]; a uniform space (X, U) is called (strongly) uniformly u -paracompact if every uniform cover of (X, U) has a locally (star) finite uniform refinement [4]; a cover α of the uniform space (X, U) is called uniformly locally finite, if there exists a uniform cover, each of whose members meets at most finitely many members of α [8]; a uniform space (X, U) is called strongly uniformly R -paracompact if every open cover of (X, U) has a uniformly locally (star) finite open refinement [5]; a cover α of the uniform space (X, U) is called uniformly locally (star) finite, if there exists a uniform cover $\beta \in U$ such that every $B(\alpha(B))$ meets at most finitely many members of α [5]; a uniform space (X, U) is called uniformly $P(B)$ -paracompact in the sense of Pasyukov (Borubaev), if, for every (finitely-additive) open cover α of (X, U) , there exists a sequence $\{\alpha_n : n \in \mathbb{N}\} \subset U$ such that for every $x \in X$, there exists $n \in \mathbb{N}$ and $A \in \alpha$ such that $\alpha_n(x) \subset A$ (UP) [3]; a uniform space (X, U) is called strongly uniformly $P(B)$ -paracompact if it is uniformly $P(B)$ -paracompact and the topological space (X, τ_U) is strongly paracompact [5]; a uniform space (X, U) is called uniformly F -paracompact in the sense of Frolik if every open cover of (X, U) has a uniformly σ -discrete (i.e. it is decomposed into a union of a countable number of uniformly discrete subsystems) open refinement [9]; a uniform space (X, U) is called strongly uniformly F -paracompact if it is uniformly F -paracompact and the topological space (X, τ_U) is strongly paracompact [5]; a uniform space (X, U) is called uniformly paracompact if every open cover of (X, U) has a uniformly σ -locally finite (i.e. it is decomposed into a union of countably many uniformly locally finite subsystems) open refinement [3]; a uniform space (X, U) is called strongly uniformly paracompact if every open cover of (X, U) has a uniformly σ -star finite (i.e. it is decomposed into a union of countably many uniformly star finite subsystems) open refinement [5]; a uniform space (X, U) is called uniformly Menger if for each sequence $\{\alpha_n : n \in \mathbb{N}\} \subset U$ there is a sequence $\{\beta_n : n \in \mathbb{N}\}$ such that for each $n \in \mathbb{N}$, β_n is a finite subset of α_n and $X = \bigcup_{n \in \mathbb{N}} \beta_n$ [6]; a uniform space (X, U) is called uniformly Hurewicz if for each sequence $\{\alpha_n : n \in \mathbb{N}\} \subset U$ there is a sequence $\{\beta_n : n \in \mathbb{N}\}$ such that for each $n \in \mathbb{N}$, β_n is a finite subset of α_n and for each $x \in X$ we

have $x \in \bigcup \beta_n$ for all but finitely many $n \in \mathbb{N}$ [6]; a uniform space (X, U) is called a uniform γ -sets if for each sequence $\{\alpha_n : n \in \mathbb{N}\} \subset U$ there is a sequence $\{x_n : n \in \mathbb{N}\}$ such that each x_n is contained in $\text{St}(x_n, \alpha_n)$ for all but finitely many $n \in \mathbb{N}$ [6]; a uniform space (X, U) is called uniformly Rothberger if for each sequence $\{\alpha_n : n \in \mathbb{N}\} \subset U$ there is a sequence $\{B_n : n \in \mathbb{N}\}$ such that for each $n \in \mathbb{N}$, $B_n \in \alpha_n$ and $X = \bigcup_{n \in \mathbb{N}} B_n$ [6]; a uniform space (X, U) is called uniformly I -Lindelöf if every uniform cover admits a countable uniform refinement [4]; a uniform space (X, U) is called uniformly A -Lindelöf in the sense of Aparina if for every open cover α of (X, U) there exists $\gamma \in U$ and a countable open cover $\beta = \{B_n : n \in \mathbb{N}\}$ such that $\gamma \succ \alpha^\wedge$ and $\gamma([B_n]_{\tau_U}) \subset B_{n+1}$ for any $n \in \mathbb{N}$, where $[B_n]_{\tau_U}$ is the closure of B_n in the topology τ_U induced by the uniformity U [1]; a uniform space (X, U) is called uniformly B -Lindelöf in the sense of Borubaev if it is uniformly B -paracompact and uniformly I -Lindelöf [2]; a uniform space (X, U) is called uniformly K -Lindelöf in the sense of Kanetov if it is uniformly R -paracompact and uniformly I -Lindelöf [5]; a uniform space (X, U) is called metrizable if there exists a metric ρ on X generating the uniformity U [2]: let $H \subset U$ be any system of uniform covers, a filter F is called Cauchy H -filter in (X, U) if $\alpha \cap F \neq \emptyset$ for any $\alpha \in H$; a uniform space (X, U) is called H -complete if every Cauchy H -filter has at least one condensation point, i.e. exists a point $x \in \bigcap \{[B] : B \in F\} \neq \emptyset$; the least cardinal number τ is called the index of completeness of (X, U) if there is a system $H \subset U$ such that $|H| = \tau$ and (X, U) is H -complete, and denoted by $ic(U) \leq \tau$ [2]; a uniform space (X, U) is called uniformly complete in the sense of Čech, if $ic(U) \leq \aleph_0$ [2]; a uniform space (X, U) is called uniformly plume if there is a countable pseudouniformity $P \subset U$ such that $\bigcap \{\alpha(x) : \alpha \in P\} = K_x$ is compact for all $x \in X$ and the system $\{\alpha(K) : \alpha \in P\}$ is a fundamental system of neighborhoods of the compact K_x in (X, τ_U) for each $x \in X$ [2]; a uniform space (X, U) is called uniformly connected if the uniformity U does not contain disjunctive covers consisting of no more than one element [2]; a uniform space (X, U) is called uniformly chained if for each cover $\alpha \in U$ there is a positive integer n such that for every pair of points $x, y \in X$ one can choose a chained sequence $\{A_1, A_2, \dots, A_k\} \subset \alpha$ such that $k \leq n$, $x \in A_1$ and $y \in A_k$. [2]; a uniform space (X, U) is called uniformly pseudocompact if every uniformly continuous real function on (X, U) is bounded [2]; we say that the dimension of a uniform space (X, U) does not exceed n , and write that $\dim(X, U) \leq n$, if the uniformity U has a base B consisting of covers of order $\leq n + 1$ [2].

2. Uniformly Hattori Space

Let $\mathbb{R}$ be the set of real numbers and $A \subset \mathbb{R}$. Put $B_A = \{\alpha_\varepsilon : \varepsilon > 0\}$, where $\alpha_\varepsilon = \{(x - \varepsilon, x + \varepsilon) : x \in A\}, \{[x, x + \varepsilon) : x \in \mathbb{R} \setminus A\}$.

Theorem 2.1. *The family $B_A = \{\alpha_\varepsilon : \varepsilon > 0\}$ of covers forms a base of a uniformity on $\mathbb{R}$.*

Proof. Let us check that conditions (B1)–(B3) are satisfied. Let $\alpha_{\varepsilon_1}, \alpha_{\varepsilon_2} \in B_A$. Then $\alpha_\varepsilon \succ \alpha_{\varepsilon_1}$ and $\alpha_\varepsilon \succ \alpha_{\varepsilon_2}$, where $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$. Let $\alpha_\varepsilon \in B_A$ be an arbitrary cover. We show that the cover $\alpha_{\varepsilon/3}$ is a strongly star refinement of the cover α_ε . To do this it is enough to show that

- 1) For any point $x \in A$ the inclusion $\alpha_{\varepsilon/3}((x - \frac{\varepsilon}{3}, x + \frac{\varepsilon}{3})) \subset (x - \varepsilon, x + \varepsilon)$ holds;
- 2) For any point $x \in \mathbb{R} \setminus A$ the inclusion $\alpha_{\varepsilon/3}([x, x + \frac{\varepsilon}{3})) \subset [x, x + \varepsilon)$ holds.

Let $y \in \alpha_{\varepsilon/3}((x - \frac{\varepsilon}{3}, x + \frac{\varepsilon}{3}))$ be an arbitrary point. Then there exists $t \in \mathbb{R}$ such that $(t - \frac{\varepsilon}{3}, t + \frac{\varepsilon}{3}) \cap (x - \frac{\varepsilon}{3}, x + \frac{\varepsilon}{3}) \neq \emptyset$ and $y \in (t - \frac{\varepsilon}{3}, t + \frac{\varepsilon}{3})$. Let $z \in (t - \frac{\varepsilon}{3}, t + \frac{\varepsilon}{3}) \cap (x - \frac{\varepsilon}{3}, x + \frac{\varepsilon}{3})$. Then $|x - y| \leq |x - t| + |t - y| \leq |x - z| + |z - t| + |t - y| \leq \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$. Thus,

$y \in (x - \varepsilon, x + \varepsilon)$. Let $y \in \alpha_{\varepsilon/3}([x, x + \frac{\varepsilon}{3}))$ be an arbitrary point. Then there exist $a \in X$ such that $[a, a + \frac{\varepsilon}{3}) \cap [x, x + \frac{\varepsilon}{3}) \neq \emptyset$ and $y \in [a, a + \frac{\varepsilon}{3})$. Let $b \in [a, a + \frac{\varepsilon}{3}) \cap [x, x + \frac{\varepsilon}{3})$. Then $|x - y| \leq |x - a| + |a - y| \leq |x - b| + |b - a| + |a - y| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$. Thus, $y \in [x, x + \varepsilon)$.

Let $x, y \in \mathbb{R}$ be arbitrary points and $x \neq y$. If $x, y \in A$, then there exists $\varepsilon > 0$ such that $y \notin \alpha_{\frac{\varepsilon}{2}}(x)$ and $x \notin \alpha_{\frac{\varepsilon}{2}}(y)$, where $\varepsilon = |x - y|$. If $x, y \in \mathbb{R} \setminus A$ or at least one of them belongs to the set $\mathbb{R} \setminus A$, then there exists $\varepsilon > 0$ such that $y \notin \alpha_{\varepsilon}(x)$ and $x \notin \alpha_{\varepsilon}(y)$, where $\varepsilon = |x - y|$. Consequently, $\cap\{\alpha_{\varepsilon}(x) : \varepsilon > 0\} = \{x\}$ for any $x \in \mathbb{R}$. $\square$

By U_A we denote the family of all covers of $\mathbb{R}$ refined by a cover from B_A . It is easy to see that the family U_A of covers of $\mathbb{R}$ is a uniformity on $\mathbb{R}$.

We will call this uniformity the Hattori uniformity and the uniform space $(\mathbb{R}, U_A)$ is a uniform Hattori space.

If $A = \mathbb{R}$, then the uniformity U_A coincides with the natural uniformity, and if $A = \emptyset$, then the uniformity U_A coincides with the Sorgenfrey uniformity.

The following theorem shows how the Hattori uniformity U_A generates the Hattori topology τ_A on $\mathbb{R}$, [10].

Theorem 2.2. *For the Hattori uniformity U_A on $\mathbb{R}$ the family $\tau_{U_A} = \{O \subset \mathbb{R} : \text{for each } x \in O$*

there exists $\varepsilon > 0$ such that $\alpha_{\varepsilon}(x) \subset O\}$ forms a Hattori topology on $\mathbb{R}$ and the topological space $(\mathbb{R}, \tau_{U_A})$ is a T_1 -space and $\tau_{U_A} = \tau_A$.

Proof. We prove that the family τ_{U_A} forms a topology on X . To do this, we need to check that all the axioms of topology are satisfied. From the definition of the family τ_{U_A} it follows that $\emptyset \in \tau_{U_A}$. Let $x \in \mathbb{R}$ be an arbitrary point. If $x \in A$, then $O = (x - \varepsilon, x + \varepsilon)$; if $x \in \mathbb{R} \setminus A$, then $O = [x, x + \varepsilon)$. Then for any $\varepsilon > 0$, $\alpha_{\varepsilon}(x) \subset \mathbb{R}$. Hence $\mathbb{R} \in \tau_{U_A}$. Let $O_1, O_2 \in \tau_{U_A}$ and $x \in O_1 \cap O_2$. If $x \in A$, then $O_1 = (x - \varepsilon', x + \varepsilon')$ and $O_2 = (x - \varepsilon'', x + \varepsilon'')$, where $\varepsilon', \varepsilon'' > 0$; if $x \in \mathbb{R} \setminus A$, then $O_1 = [x, x + \varepsilon^*)$ and $O_2 = [x, x + \varepsilon^{**})$, where $\varepsilon^*, \varepsilon^{**} > 0$. Then from the definition of τ_{U_A} it follows that there exists $\varepsilon_1, \varepsilon_2 > 0$ such that $\alpha_{\varepsilon_1}(x) \subset O_1$ and $\alpha_{\varepsilon_2}(x) \subset O_2$. By condition (P1) there exists a uniform cover $\alpha_{\varepsilon} \in U$ such that $\alpha_{\varepsilon} \succ \alpha_{\varepsilon_1}$ and $\alpha_{\varepsilon} \succ \alpha_{\varepsilon_2}$. It follows that $\alpha_{\varepsilon}(x) \subset \alpha_{\varepsilon_1}(x) \cap \alpha_{\varepsilon_2}(x) \subset O_1 \cap O_2$. Therefore, $O_1 \cap O_2 \in \tau_{U_A}$, where $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$. Let $\lambda \subset \tau_{U_A}$ be an arbitrary subfamily. We prove that $\cup \lambda \in \tau_{U_A}$. Let $x \in \cup \lambda$ be an arbitrary point. Then $x \in O \subset \cup \lambda$. If $x \in A$, then $O = (x - \varepsilon, x + \varepsilon)$, and if $x \in \mathbb{R} \setminus A$, then $O = [x, x + \varepsilon)$. Since $O \in \tau_{U_A}$, then there exists $\varepsilon > 0$ such that $\alpha_{\varepsilon}(x) \subset O \subset \cup \lambda$. Therefore, $\cup \lambda \in \tau_{U_A}$. Hence, the family τ_{U_A} forms a topology on $\mathbb{R}$.

Now we prove that $(\mathbb{R}, \tau_{U_A})$ is a T_1 -space. To do this, we show that $O = \mathbb{R} \setminus \{x\}$ is an open set. If $x \in A$, then $O = (x - \varepsilon, x + \varepsilon)$; if $x \in \mathbb{R} \setminus A$, then $O = [x, x + \varepsilon)$.

By condition (P4) there exists $\varepsilon > 0$ such that no element of the cover $\alpha_{\varepsilon} \in U$ contains both x and $z \in \mathbb{R} \setminus \{x\}$, i.e. $z \notin \alpha_{\varepsilon}(x)$. Since $\alpha_{\varepsilon}(z) \subset O$ is open, then the set $O = \mathbb{R} \setminus \{x\}$ is open. Consequently, the one point set $\{x\}$ is closed. $\square$

Remark 2.1. Hattori's uniformities can be easily defined in terms of entourages of the diagonal.

Problems:

1. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ precompact? – The answer is negative;
2. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ compact? – The answer is negative;
3. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ τ -bounded?
4. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ complete?
5. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ complete in Dieudonne sense?
6. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly locally compact?

7. What is the weight of a uniformly Hattori space?
8. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ sequentially complete?
9. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly paracompact in the sense of Rice?
10. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly paracompact in the sense of Pasynkov?
11. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly paracompact in the sense of Borubaev?
12. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly paracompact in the sense of Frolik?
13. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly paracompact in the sense of Aparina?
14. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly Lindelöf in the sense of Borubaev?
15. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly Lindelöf in the sense of Kanetov?
16. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly Lindelöf in the sense of Aparina?
17. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ strongly uniformly R -paracompact?
18. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ strongly uniformly A -paracompact?
19. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ strongly uniformly P -paracompact?
20. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ strongly uniformly B -paracompact?
21. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ strongly uniformly F -paracompact?
22. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ strongly uniformly paracompact?
23. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ (strongly) uniformly u -paracompact?
24. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly connected?
25. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly chained?
26. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly plume?
27. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly complete in the sense of Čech?
28. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ uniformly pseudocompact?
29. Is a uniformly Hattori space $(\mathbb{R}, U_A)$ metrizable?
30. Does a uniformly Hattori space $(\mathbb{R}, U_A)$ have the uniform Menger property?
31. Does a uniformly Hattori space $(\mathbb{R}, U_A)$ have the uniform Hurewicz property?
32. Does a uniformly Hattori space $(\mathbb{R}, U_A)$ have the uniform Rothberger property?
33. Does a uniformly Hattori space $(\mathbb{R}, U_A)$ have the uniform γ -set property?
34. Is every subspace of a uniformly Hattori space is a uniformly Hattori space?
35. What is the uniform dimension $(\dim(\mathbb{R}, U_A))$ of a uniformly Hattori space?

3. Conflict of interests

The authors declare that have no conflicts of interest.

4. Acknowledgments

The corrections and excellent suggestions of the anonymous reviewer are gratefully acknowledged.

References

- [1] L.V. Aparina, *Uniform Lindelöf spaces*, Trudy Mosk. Math. Obshestva, **57** (1996), 3–15.
- [2] A.A. Borubaev, *Uniform Topology and its Applications*, Bishkek: Ilim, (2021).
- [3] D. Buhagiar, B.A. Pasynkov, *On uniform paracompactness*, Czech. Math. J., **46** (1996), 577–586.

- [4] B.E. Kanetov, *Some Classes of Uniform Spaces and Uniformly Continuous Mappings*, Bishkek: Jusup Balasagyn KNU, (2013).
- [5] B.E. Kanetov, *Uniform Analogues of Strongly Paracompact Spaces*, Bishkek: Jusup Balasagyn KNU, (2026).
- [6] Lj.D.R. Kočinac, *Selection principles in uniform spaces*, Note Mat., **22** (2003), 127–139.
- [7] J. Isbell, *Uniform space*, Providence, (1964).
- [8] M.D. Rice, *A note on uniform paracompactness*, Proc. Amer. Math. Soc., **62** (1977), 359–362.
- [9] Z. Frolik, *On uniform paracompact spaces*, Czech. Math., J. **33** (1983), 476–484.
- [10] Y. Hattori, *Order and topological structures of posets of the formal balls on metric spaces*, Mem. Fac. Sci. Eng. Shimane Univ. Series B: Mathematical Science, **43** (2010), 13–26.